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STUDY OF OBJECTS MOVEMENT BY DYNAMICAL PHOTOGRAMMETRY METHODS

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One of the basic problems of study of moving objects by photogrammetrical methods is a defenition of velocities and trajectories of movement of their characteristic points. In this case the usual method of solving consists in finding space coordinates of points in the moments of camera expositions and in further their approximation by curves which are determined as trajectories. The velocities are found by segments of way pathed by points and by time intervals between the camera expositions. In so doing methods of classical photogrammetry are used where the picture is regarded as momentary central projection of objects space, that is identical to projection which would be obtained under condition that during the exposition both camera and photographed objects are mutually immobile. Such an assumption is appropriate when velocities of movement are not significant and photographing is done by the camera having diaphragm shutter. If it is necessary to compensate image remove and cameras are used where the picture is exposed not in an instant then it is reasonable to solve the problem by methods of dynamical photogrammetry [1] which is the development of classical problem. The dynamical photogrammetry supplements the classical one and allows to regard in a new way the phenomena which can't be investigated from the static conceptions. It proceeds from the fact that photographing is carried out in the dynamical conditions and takes into account mutual movement of both the projecting system of camera and subjects in objects space.

The dynamical photogrammetry is based on the differential equations of motion of optical image points in camera. By their integration it is possible to obtain equations of trajectories of optical image points including a time parameter "t". The trajectories of movement of investigated subjects are the prototype of these trajectories in objects space. Therefore the dynamical photogrammetry establishes a direct relation between the subject image on a photo and parameters of its movement in nature. In [1] the example of using of the dynamical photogrammetry dependencies is given for precise phototriangulation on the base of aerial pictures taken by camera having a remove compensation system and curtain slotted shutter. Such pictures are geometrically distorted because of the fact that in the different parts various phases of moving optical image are fixed due to the unsimultaneity of photographic exposure. Parametric equations of trajectories of optical image points permit to take into account the distortions in the coordinates of picture points. They are also necessary for analysis and synthesis of the image remove compensation system. Classical photogrammetry does not solve such problems at all. But in many other cases also when traditional methods of classical photogrammetry are used for study of moving objects the dynamical photogrammetry apparatus turns to be more preferable. Some examples may serve as illustrations.

Let's consider the most common case when the optical axis of camera closely coincides with a normal to the midplane of the objects movement. For such a case the parametric equations of trajectories of optical image points may be written as follows :

$$\frac{x - x_1}{t} = \frac{x_1 y_1}{f}\,\omega_x + \left(f + \frac{x_1^2}{f}\right)\omega_y - y_1\,\omega_z - \frac{W_z}{H}\,x_1 + $$

$$+ \frac{f}{H}\left\{W_x\left[1 + 2\,\frac{\alpha_y}{f}\,x_1 - \frac{\alpha_x}{f}\,y_1 + \frac{f}{H}\,t\left(\frac{\alpha_y}{f}\,W_x - \frac{\alpha_x}{f}\,W_y\right)\right] - \frac{\alpha_x}{f}\,x_1\,W_y\right\} \qquad (1)$$

$$\frac{y - y_1}{t} = \left(f + \frac{y_1^2}{f}\right)\omega_x + \frac{x_1 y_1}{f}\,\omega_y + x_1\,\omega_z - \frac{W_z}{H}\,y_1 + $$

$$+ \frac{f}{H}\left\{W_y\left[1 + \frac{\alpha_y}{f}\,x_1 - 2\,\frac{\alpha_x}{f}\,y_1 + \frac{f}{H}\,t\left(\frac{\alpha_y}{f}\,W_x - \frac{\alpha_x}{f}\,W_y\right)\right] + \frac{\alpha_y}{f}\,y_1\,W_x\right\} \qquad (2)$$

where x , y are current coordinates of image point in OXY system, being realized by coordinate marks on picture aperture of camera ;

x_1, y_1 are coordinates of the same point in the initial phase of designing ;

t is a current time being counted off from the initial phase ;

f is a focal length of camera ;

$\omega_x , \omega_y , \omega_z$ are angular velocities of rotation about axes ox, oy and principal beam of the objective (axis oz), respectively ;

H is an initial distance (height) of projection center over the point of object, the optical image coordinates of which are x, y ;

W_x , W_y , W_z are linear velocities of mutual displacement of both object points and camera in the direction of the axes OX, OY, OZ of the OXYZ system which is a prototype of the oxyz system in the objects space ;

α_x , α_y are inclination angles of camera in the initial phase of projecting.

Usually, the movement of objects are studied with reference to the net of fixed datum points the position of which is initially known in OXYZ the system. Then, the values of H, α_x , α_y get meaning of elements of exterior orientation of camera in the initial phase, and $\omega_x t_u = \Delta \omega$, $\omega_y t_u = \Delta \alpha$, $\omega_z t_u = \Delta \kappa$ are elements of internal orientation of pair of plates where $t_u = t$ is a photographing interval.

From the dynamical photogrammetry point of view the overlapping photographs forming a stereopair ought to be regarded as different phases of optical dynamic projection fixed in photographic way in some time interval.

Having a series of overlapping pictures of an object to be investigated it is possible to get values x_1 , x and y_1 , y of its separate points by the following equations :

$$\left.\begin{array}{ll} x_1 = x_{\varphi 1} - \sigma x_1 - x_o & x = x_\varphi - \sigma x - x_o \\ y_1 = y_{\varphi 1} - \sigma y_1 - y_o & y = y_\varphi - \sigma y - y_o \end{array}\right\} \quad (3)$$

where $x_{\varphi 1}$, $y_{\varphi 1}$, x_φ, y_φ are coordinates of points of the consecutive pair in the $O_\varphi x_\varphi y_\varphi$system, realised by collimating marks fixed in the pictures ; σx_1 , σx, σy_1 , σy are summary corrections for curvilinear distortion of the objective, deformation and flattering of photographic material ; x_o, y_o are coordinates of the optical image center. If H, α_x, α_y, $\Delta\alpha$, $\Delta\omega$,$\Delta\kappa$ are known (are previously measured or determined by points of control in a regular manner used in classical photogrammetry) and in the process of photographing the exposition moments were exactly fixed, then in Eqs. (1), (2) three unknown values W_x, W_y, W_z are left. Taking three sequential pictures it is possible for the points which are in a zone of triple overlap to compose a system of three pairs of equations of types (1) and (2). Solving this system we shall get values and the estimate of accuracy of definition of average velocities W_x, W_y, W_z in the time interval between the expositions of the first and third pictures.

Sequentially processing the pictures of whole series it is possible to obtain in similar manner several velocities of object points during the experiment. Having carried out numerical integration of these velocities we shall determined the trajectories of points.

If the optical axis of camera is considerably deviated from the normal to the given midplane (oblique photography), the problem can be solved according to the same dependencies (1), (2), but by means of successive approximation . Let's consider two practical examples of the dynamical photogrammetry dependencies using them for objects movement study.

In the hydrological investigations velocities and trajectories of surface streams in water bodies are determined by the air survey technique using radio altimeter. This technique consists in photographing of the freely swimming marking subjects (ice-floes, targets, coloured pots and so on) simultaneously fixing in the pictures the waterside line. For the detailed study of streams it is necessary to take pictures in short time intervals, that is with big overlap. Synchronously with the camera expositions the readings of both chronometer and radio altimeter (R.A.) are registered.

As a result of photogrammetrical processing at which the pictures are graphicaly or opticaly transformed, a position plan of the marking objects (M.O.) is obtained in the moments of expositions and the velocities of streams are calculated according to both the path passed by M.O. and time intervals [2]. This problem can be solved by the analytic methods of dynamical photogrammetry and the essentially higher accuracy can be obtained.

In this case in the expressions (1), (2) $x - x_1 = p$, $y - y_1 = q$ are horizontal and vertical parallaxes of M.O. images on the neighbouring pictures ; $\Delta H = W_z t_u = H - H_1$; H and H_1 are the altimetric points above the water surface of the subsequent and preceding pictures of the consecutive photographs, determined by R.A. readings.

The mutual inclination angles and the mutual turn angles as well as the absolute inclination angles of the first picture can be found by methods of classical photogrammetry according to parallaxed points of detail on the water levels. Seeing that they all are located on the same plane and the altimetric point is known from R.A. readings ; these angles can be determined without the definition of the points coordinates in nature. Then, the x_φ, y_φ coordinates of M.O. images are measured on the first and second pictures and the oxy system are determined by Eq. (3). Next, M.O. parallaxes are defined and Eqs. (1), (2) are solved relative to W_x and W_y that are stream velocities being of our interest. Inasmuch as Eqs. (1), (2) are not linear it is reasonable to linearize them, substituting

$$W_x = \widetilde{W}_x + \Delta W_x$$

$$W_y = \widetilde{W}_y + \Delta W_y$$

where $\widetilde{W}_x = \frac{pH}{f\,t_u}$; $\widetilde{W}_y = \frac{qH}{f\,t_u}$

are approximate values of stream velocities ; ΔW_x and ΔW_y are their corrections. The trajectories of streams can be found by W_x and W_y integrating.

The parametrical equations (1), (2) can be used for constructing the mathematical model of the process of projecting locality which is an airpicture. With its help it is convenient to investigate, for instance, the dynamic of survey aircraft movements. In this case it is reasonable to get for this model the following aspect :

$$x_\varphi - \sigma x - x_0 = X\frac{f}{H}\left(1 + \frac{X}{H}\alpha_y - \frac{Y}{H}\alpha_x\right) + \frac{W}{H}f.t + \frac{f}{H^2}\Big[W_H X + + W_G(2X\alpha_y - Y\alpha_x + W_G t\alpha_y) + XY\omega_x + (H^2 + X^2)\omega_y - HY\omega_z\Big]t \qquad (4)$$

$$y_\varphi - \sigma y - y_0 = Y\frac{f}{H}\left(1 + \frac{X}{H}\alpha_y - \frac{Y}{H}\alpha_x\right) + \frac{f}{H^2}\Big[W_H Y + W_G(Hi + Y\alpha_y) + + (H^2 + Y^2)\omega_x + XY\omega_Y + HX\omega_z\Big]t \qquad (5)$$

Here : X, Y are coordinates of point of detail of the given OXYZ system is a prototype of some image point with the coordinates x_φ, y_φ , in the initial phase ; W_G, W_H are horizontal (itinerary) and vertical velocities of aircraft ; i is a drift angle or an angle between vector W_G and axis OX (it is meant that camera is oriented in such a way that its abscissa axis gets the direction of flight). The parameters of aircraft movement that are of our interest are W_G, W_H, i, ω_x, ω_y, ω_z. In so doing we shall consider that the camera is well fixed, that is it repeats all movements of the aircraft.

Let the first picture of the stereopair be the initial phase for which $t = 0$ and the flight is taken place above the proving ground with a rather detail network of control points that are identifiable on the pictures with the coordinates known in $O_0 X_0 Y_0 Z_0$ some system.

Then, it is possible to find the inclination angles of the first picture α_x, α_y, coordinates of control points in its OXYZ system and their altimetric points H by solving the usual photographic intersection.

Let's take a number of points of control according to the number of the above-mentioned parameters of flight to be determined, identify them on the second picture and measure their coordinates x_φ, y_φ. Next, the system of Eqs. (4), (5) is formed, substituting in their right part H, X, Y of each point and α_x, α_y of the first picture found from the solution of the intersection. The interval of the photographing t is substituted instead of t_u. Let's replace in (4), (5),

$$W_G = \widetilde{W}_G + \Delta W$$

$$\widetilde{W}_G = \frac{H}{f.t_u} \sum_1^n (x_\varphi - x_{\varphi_1})$$

where $\widetilde{W}_G$ is an approximate value of W_G;

n is the number of control points, and then these equations are linearized excluding those terms that contain the products $\Delta W_G \alpha_x$, $\Delta W_G \alpha_y$, $\Delta W_G i$ because of their unsignificance. By solving the system of linear equations the average values of W_G, W_H, i, ω_x, ω_y, ω_z can be obtained for the photographing interval t_u. The redundant number of control points usually raises the accuracy and allows to get its estimation.

References

[1] B.N. Rodionov
"On Dynamical Photogrammetry" - the report at the Photogrammetric Symposium. Moscow, 1968.

[2] B.N. Rodionov
"Photogrammetrical Works at Airsurvey for Defenition of Directions and Velocities of River Streams". Transactions of MIIGAiK. 1957, number 27.

Abstract

Some aspects of study of moving objects by the methods of dynamical photogrammetry which allow for the movement of both the projecting system and subjects in object spaces are discussed. By integrating the differential equations of the motion of optical image points in the camera one can obtain the equations of trajectories of optical image points with respect to a time parameter "t". The paper gives several cases of application of the dynamical photogrammetry with their appropriate relationships.